UNSTEADY AND STEADY AERODYNAMIC FORCES OF SLENDER DELTA WINGS ACCORDING TO NEWTONIAN THEORY*

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Abstract

The problems of hypersonic flow past a stationary and an oscillating conical delta wing at high angles of attack with detached shock waves are studied using the thin shock layer theory. In the stationary wing case an accurate perturbation solution is obtained thus improving and extending Messiter's and other existing theories for flat and curved wings. For the pitching delta wings with small amplitude, simple analytic formulae are derived for the aerodynamic derivatives. The effects of wing curvature on its stability were studied systematically and it is found that the pitching motion of a hypersonic delta wing is always stable aerodynamically.

I. Introduction

With the advent of the space shuttle and high-performance modern military aircrafts, information on steady and unsteady aerodynamic forces, and in particular on dynamic stability of vehicles at high angles of attack are needed at all Mach number range. This is most evident in a recent survey of Orlik-Ruckemann(1) who also showed that at hypersonic speed the damping-in-pitch derivative is one of the important information required. Whilst Orlik-Ruckemann's survey reveals an almost total lack of wind tunnel capabilities for measuring dynamic derivatives at high angles of attack, very little theoretical work exists for predicting such derivatives.

In the case of two-dimensional pitching wedges with attached shock waves, exact inviscid flow theory of Hui(2) predicts dynamic instability for large wedge angles, or equivalently for flat plate at high angles of attack. These theoretical predictions were confirmed experimentally.

By contrast, very little theoretical work was done on three-dimensional problems. One of the most important, and perhaps the simplest, unsteady three-dimensional problem is that of a conical delta wing performing hormonic pitching motion in a supersonic stream (Fig. 1). Depending on the combination of the free stream Mach number M_∞, the mean angle of attack α, the specific heat ratio γ of the gas, and the wing geometry (such as the aspect ratio b and the cross-sectional shape) the shock wave developed on the lower surface may be attached to or detached from the leading edges. Whilst the attached shock case was studied by Liu and Hui(3), the present paper is concerned with the case when the shock is detached from the leading edges but is still attached to the wing apex.

The gas is assumed non-viscous, non-heat conducting with constant specific heats. The amplitude of oscillation λ_0 is assumed small so that the unsteady flow may be regarded as a small perturbation of the steady flow past the wing at its mean position. Furthermore the hypersonic thin shock layer approximation will be made in which the parameter characterising the thickness of the shock layer

$$\epsilon = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_\infty^2 \sin^2\alpha} \qquad (1)$$

is much less than 1.0 so that terms $O(\epsilon^2)$ can be neglected.

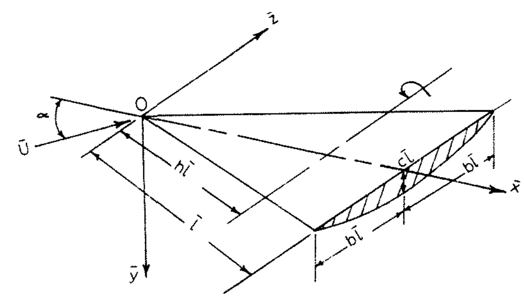

Fig.1 Wing geometry and coordinate system.

As in any perturbation theory, the accuracy of the unsteady flow calculation depends crucially on the availability and accuracy of the corresponding reference steady flow solution. For flat delta wings, the steady flow problem was tackled by Messiter(4) using thin shock layer approximation. In sections 2 and 3, Messiter's thin shock layer solution will be improved and extended to general conical delta wing cases. This solution will then be used in section 4 to derive closed-form solution for the unsteady pressure and the stability derivatives of the wing.

II. Formulation of the Problem

Consider a convex conical delta wing (Fig. 1) performing symmetrically harmonic pitching motion in a hypersonic stream of gas. A cartesian coordinate system $\bar{o}\bar{x}\bar{y}\bar{z}$, fixed in space, is chosen such that the plane of leading edges of the wing in its mean position lies in the plane $\bar{y} = 0$. Let the amplitude λ_0 of oscillation be small and the pitching axis in the leading edge plane parallel to $\bar{z}$-axis and at a distance $h\bar{\ell}$ from the wing apex, $\bar{\ell}$ being the length of the wing. When terms $O(\lambda_0^2)$ are neglected, the instantaneous position of the wing surface is given by

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$$B(\bar{x},\bar{y},\bar{z},\bar{t}) \equiv \bar{y} - \bar{y}_b(\bar{x},\bar{z}) - \lambda_o e^{i\omega\bar{t}}(h\bar{\ell} - \bar{x}) = 0 \qquad (2)$$

where $\bar{y}_b(\bar{x},\bar{z})$ is the wing equation at its mean position, ω the circular frequency and $\bar{t}$ the time variable. The flow region of interest is that on the lower surface of the wing bounded by the shock wave and the planes $\bar{z} = \pm b\bar{x}$, where the aspect ratio $b = \bar{b}/\bar{\ell}$, $\bar{b}$ being the semi-span of the wing. At hypersonic speed the upper surface flow has very little contribution to the aerodynamic forces and will be neglected. Our aim is to find the steady and unsteady pressure distributions on the lower surface and hence the normal force coefficeint and the stability derivatives.

Based on the assumptions stated in the introduction, the governing equations of continuity, momentum, and energy for the pressure $\bar{p}$, density $\bar{\rho}$ and the components $\bar{u}$, $\bar{v}$, $\bar{w}$ of the velocity $\vec{q}$ may be written in the form

$$\bar{\rho}_{\bar{t}} + \nabla \cdot (\bar{\rho}\,\vec{q}) = 0 \qquad (3a)$$

$$\vec{q}_{\bar{t}} + \vec{q} \cdot \nabla\vec{q} + \frac{1}{\bar{\rho}}\nabla\bar{p} = 0 \qquad (3b)$$

$$(\bar{p}/\bar{\rho}^{\gamma})_{\bar{t}} + \vec{q} \cdot \nabla(\bar{p}/\bar{\rho}^{\gamma}) = 0 \qquad (3c)$$

The boundary condition to be satisfied at wing surface is that the normal component of relative velocity vanishes; that is,

$$B_{\bar{t}} + \vec{q} \cdot \nabla B = 0 \,. \qquad (4)$$

The conditions to be satisfied at the shock surface are the Rankine-Hugoniot jump conditions:

$$[\bar{\rho}(S_{\bar{t}} + \vec{q} \cdot \nabla S)] = 0 \qquad (5a)$$

$$[\bar{\rho}(S_{\bar{t}} + \vec{q} \cdot \nabla S)^2 + (\nabla S)^2\bar{p}] = 0 \qquad (5b)$$

$$[\tfrac{1}{2}(S_{\bar{t}} + \vec{q} \cdot \nabla S)^2 + (\nabla S)^2 \frac{\gamma}{\gamma-1}(\bar{p}/\bar{\rho})] = 0 \qquad (5c)$$

$$[\vec{q} \times \nabla S] = 0 \qquad (5d)$$

where the square brackets denote the change in the enclosed quantity across the shock and S is the shock wave equation given by

$$S(\bar{x},\bar{y},\bar{z},\bar{t}) \equiv \bar{y} - \bar{y}_s(\bar{x},\bar{z},\bar{t}) = 0 \qquad (6)$$

$\bar{y}_s(\bar{x},\bar{z},\bar{t})$ is the shock height, as yet unknown. It has to be found as a part of the solution. The first threee of Eq.(5) are the usual conservation equations of mass, momentum, and specific enthalpy across a normal shock, the last is a vector equation (equivalent to two scalar equations) expressing continuity of tangential velocity. The system (5) is sufficient to allow calculation of the functions $\bar{u}$, $\bar{v}$, $\bar{w}$, $\bar{p}$, $\bar{\rho}$, just behind the shock surface, in terms of the unknown shock shape $\bar{y}_s$.

Following Messiter we consider the interesting case for which the wing apex angle and the Mach angle are of the same order of magnitude as $\epsilon \to 0$; this can be characterized by the constancy of the similarity parameter

$$\Omega = \frac{\bar{b}}{\bar{\ell}\,\epsilon^{1/2}\tan\alpha} \qquad (7)$$

Furthermore, we consider a family of conical wings such that the maximum thickness of wing $c\bar{\ell}$ at $\bar{x} = \bar{\ell}$ is of the same order of magnitude as ϵ in the limit $\epsilon \to o$, i.e.

$$\frac{c}{\epsilon} \to \text{const as } \epsilon \to o \,. \qquad (8)$$

Messiter[4] pointed out that the case of shock wave detached from the leading edges correspond to $0 < \Omega < 2$. From order of magnitude analysis (e.g. shock layer thickness is $o(\epsilon)$, semi-span $b = O(\sqrt{\epsilon})$) a correct scaling of the independent variables is

$$x = \frac{\bar{x}}{\bar{\ell}}\,,\ y = \frac{\bar{y}-\lambda(h\bar{\ell}-\bar{x})}{\bar{\ell}\,\epsilon\tan\alpha}\,,\ z = \frac{\bar{z}}{\bar{\ell}\,\epsilon^{1/2}\tan\alpha}$$

$$t = \frac{\bar{t}\bar{U}\cos\alpha}{\bar{\ell}}\,,\quad \lambda \equiv \lambda_o e^{ikt} \qquad (9)$$

where $\bar{U}$ is the free stream velocity and

$$k = \frac{\omega\bar{\ell}}{\bar{U}\cos\alpha} \qquad (10)$$

is the frequency parameter.

In the limit $\epsilon \to o$ the flow is the Newtonian flow. For small ϵ and λ_o, the following expansions of the flow field are suggested, as a generalization of Messiter's expansion, to oscillating conical wings:

$$\frac{\bar{u}(\bar{x},\bar{y},\bar{z},\bar{t})}{\bar{U}\cos\alpha} = 1 + \epsilon\tan^2\alpha\, u(x,y,z) + \lambda_o e^{ikt} U_1(x,y,z) + \ldots \qquad (11a)$$

$$\frac{\bar{v}(\bar{x},\bar{y},\bar{z},\bar{t})}{\bar{U}\sin\alpha} = \epsilon\, v(x,y,z) + \lambda_o e^{ikt}\Big\{\cot\alpha[ik(h-x)-1)] + \epsilon[V_1(x,y,z) - \tan\alpha\, u(x,y,z)]\Big\} \qquad (11b)$$

$$\frac{\bar{w}(\bar{x},\bar{y},\bar{z},\bar{t})}{\bar{U}\sin\alpha} = \epsilon^{1/2} w(x,y,z) + \lambda_o\,\epsilon^{1/2} e^{ikt} W_1(x,y,z) + \ldots \qquad (11c)$$

$$\frac{\bar{p}(\bar{x},\bar{y},\bar{z},\bar{t})-\bar{p}_\infty}{\bar{\rho}_\infty\bar{U}^2\sin^2\alpha} = 1 + \epsilon\, p(x,y,z) + \lambda_o e^{ikt} P_1(x,y,z) + \ldots \qquad (11d)$$

$$\frac{\bar{\rho}_\infty}{\bar{\rho}(\bar{x},\bar{y},\bar{z},\bar{t})} = \epsilon - \epsilon^2\rho(x,y,z) - \lambda_o\,\epsilon\, e^{ikt} R_1(x,y,z) + \ldots \qquad (11e)$$

$$\bar{y}_s(\bar{x},\bar{z},\bar{t}) = \lambda_0 e^{ikt}(h\bar{\ell}-\bar{x}) + \bar{\ell}\,\epsilon\,\tan\alpha[y_s(x,z) + + \lambda_0\, e^{ikt}\, Y_1(x,z)] + \ldots \tag{11f}$$

It should be pointed out that although the two small parameters ϵ and λ_0 are generally independent, the limiting process in (11) must be understood as $\lim_{\epsilon\to 0}\lim_{\lambda_0\to 0}$ and, as shown in Ref. 5, this correctly corresponds to the definition of stability derivatives.

Substituting (9)-(11) into Eqs.(3)-(6) and equating like terms in ϵ and λ_0, we obtain the following systems of equations for the determination of the perturbation flow quantities. Thus, to linear order terms in ϵ, we have

(I)

$$v_y + w_z = 0 \tag{12a}$$

$$u_x + v\,u_y + w\,u_z = 0 \tag{12b}$$

$$v_x + v\,v_y + w\,v_z + p_y = 0 \tag{12c}$$

$$w_x + v\,w_y + w\,w_z = 0 \tag{12d}$$

$$\left(\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(p - \gamma\rho) = 0 \tag{12e}$$

at $y = y_b(x,z)$,

$$v = \left(\frac{\partial y_b}{\partial x} + w\frac{\partial y_b}{\partial z}\right) \tag{13}$$

At $y = y_s(x,z)$,

$$u = -\,y_{sx} \tag{14a}$$

$$v = y_{sx} - y_{sz}^2 - 1 \tag{14b}$$

$$w = -\,y_{sz} \tag{14c}$$

$$p = 2\,y_{sx} - y_{sz}^2 - 1 \tag{14d}$$

$$\rho = a(2\,y_{sx} - y_{sz}^2) \tag{14e}$$

At $x = 0$, $y_s = 0$ (14f)

where

$$y_b(x,z) = \frac{\bar{y}_b(\bar{x},\bar{z})}{\bar{\ell}\epsilon\,\tan\alpha} \tag{15}$$

The condition (14f) arises because the shock is assumed attached to the wing apex during the course of oscillation.

To the linear order terms in λ_0, we have [6]

$$R_{1x} + v\,R_{1y} + w\,R_{1z} + ikR_1 + U_{1x} + V_{1y} + W_{1z} = 0 \tag{16a}$$

$$U_{1x} + v\,U_{1y} + w\,U_{1z} + ikU_1 = 0 \tag{16b}$$

$$P_{1y} - \cot\alpha[k^2(h - x) + 2ik] = 0 \tag{16c}$$

$$W_{1x} + v\,W_{1y} + w\,W_{1z} + ikW_1 + P_{1z} + w_x\,U_1 + + w_y\,V_1 + w_z\,W_1 = 0 \tag{16d}$$

$$\left(\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + ik\right)(P_1 - \gamma R_1) = 0 \tag{16e}$$

(II)

at $y = y_b(x,z)$, $V_1 = U_1\dfrac{\partial y_b}{\partial x} + W_1\dfrac{\partial y_b}{\partial z}$ (17)

at $y = y_s(x,z)$, $U_1 = \tan\alpha$ (18a)

$$V_1 = \tan\alpha\; y_{sx} - 2y_{sz}\,Y_{1z} + [ik - v_y(x,y_s,z)]Y_1 + Y_{1x} + \cot\alpha(\bar{N} - y_{sz}^2)[ik(h-x)-1] \tag{18b}$$

$$W_1 = -\{w_y(x,y_s,z)\,Y_1 + Y_{1z} + \cot\alpha\; y_{sz}\cdot[ik(h-x)-1]\} \tag{18c}$$

$$P_1 = 2\cot\alpha[ik(h-x)-1] \tag{18d}$$

$$R_1 = 2a\cot\alpha[ik(h-x)-1] \tag{18e}$$

at $x = 0$, $Y_1 = 0$ (18f)

where $a = \dfrac{2}{2+N}$, $\bar{N} = \dfrac{2-N}{2+N}$, $N \equiv (\gamma-1)M_\infty^2\sin^2\alpha$ (19)

It is to be noticed that in consistance with the limiting process noted earlier, the boundary conditions in system (II) for determining the unsteady flow are satisfied at the steady mean positions of the wing (y_b) and the shock (y_s). The system (I) for the steady flow and the system (II) for the unsteady flow will be considered sucessively in sections 3 and 4. It should be noted that system (I) is non-linear whereas (II) is linear.

III. The Steady Flow Solution

In this section, we consider the solution to system (I) in section 2. For the special case of flat delta wings, $y_b = 0$, Messiter[4] reduced the problem to that of solving a single functional-differential equation. For general conical wings the wing equation may be given by

$$y_b(x,z) = x\,f(\tilde{z}), \quad \tilde{z} = z/x \tag{20}$$

By extending Messiter's method, we can also reduce the problem of solving (I) to that of the following functional-differential equation[6] for $\tilde{z}(w)$

$$\begin{cases} \dfrac{\tilde{z}'(w)\tilde{z}'[\tilde{z}(w)]}{\{\tilde{z}[\tilde{z}(w)]-\tilde{z}(w)\}^2} = \dfrac{1}{[\tilde{z}(w)-w]^2} - 1 - \\ \qquad f''[\tilde{z}(w)]\ \tilde{z}'[\tilde{z}(w)] & (21a) \\ \tilde{z}(0) = 0 & (21b) \\ \tilde{z}(1+\Omega) = \Omega & (21c) \end{cases}$$

where ( )' denotes ordinary derivative of a function.

It is convenient to make the following transformations

$$W = \frac{w}{1+\Omega} \qquad (22)$$

$$Z(W) = \frac{\tilde{z}(w)}{\Omega} \qquad (23)$$

then (21) becomes

$$\begin{cases} Z'Z'(\beta Z)(\beta Z - W)^2 = [Z(\beta Z) - Z]^2 \cdot \\ \cdot \left\{1 - \left(\dfrac{\beta Z-W}{1-\beta}\right)^2\left[1 + \beta Z'(\beta Z)f''\left(\dfrac{\beta Z}{1-\beta}\right)\right]\right\} & (24a) \\ Z(0) = 0 & (24b) \\ Z(1) = 1 & (24c) \end{cases}$$

where

$$\beta = \frac{\Omega}{1+\Omega} \qquad (25)$$

and its range for detached shock flow is $0 < \beta < \frac{2}{3}$.

Once (24) is solved the steady shock wave position y_s can be found from

$$\frac{y_s}{x} = f(\tilde{z}) + \beta \int_{\beta Z}^{W} \frac{[W^*-\beta Z(W)]Z'(W^*)}{[W^*-\beta Z(W^*)]^2}\, dW^* \qquad (26)$$

Other flow variables can also be calculated. In particular the normal force correction function F to Newtonian flow is given by

$$F = \frac{C_N - 2\sin^2\alpha - 2/\gamma M_\infty^2}{\epsilon \sin^2\alpha}$$

$$= \frac{2}{\beta}\int_0^1 \left\{\beta Z'(W)\left[\frac{2\beta W Z(W)}{(1-\beta)^2} + 2f(\Omega Z(W)) - \frac{W^2}{(1-\beta)^2} - 1\right]\right.$$

$$\left. + 2\beta^2 Z'(W) I_1 + \frac{\beta[1-(\frac{W-\beta Z(W)}{1-\beta})^2]}{[W-\beta Z(W)]^2} I_2 \right\} dW \qquad (27)$$

where

$$I_1 = \int_{\beta Z(W)}^{W} \frac{[W^*-\beta Z(W)]Z'(W^*)}{[W^*-\beta Z(W^*)]^2}\, dW^* \qquad (28a)$$

$$I_2 = \int_{\beta Z(W)}^{W} \frac{[W^*-\beta Z(W)]^3 Z'(W^*)}{[W^*-\beta Z(W^*)]^2}\, dW^* \qquad (28b)$$

For small $\beta > 0$ Eq.(24) can be solved in the series form

$$Z = Z_0(W) + \beta Z_1(W) + \beta^2 Z_2(W) + \dots , \qquad (29)$$

and the function f" , depending on wing geometry, can be expanded in the form

$$f''(\tilde{z}) = -[f_1(\tilde{z}) + \beta f_2(\tilde{z}) + \beta^2 f_3(\tilde{z}) + \dots]/\Omega, \quad (30)$$

where f_1, f_2, ... can be found once the wing geometry is given. They are assumed to be of $O(1)$ thus implying that c/b remains fixed as $\beta \to 0$. Substituting (29) and (30) in (24) and equating like powers of β we get the following systems of ordinary differential equations for the successive determination of Z_0, Z_1

$$Z_0'\, Z_0'(0)\, W^2 = Z_0^2(1 - W^2) + f_1 Z_0^1(0) W^2 Z_0^2 \qquad (31a)$$

$$Z_0(0) = 0 \qquad (31b)$$

$$Z_0(1) = 1 \qquad (31c)$$

$$-2Z_0'\, Z_0'(0) W Z_0 + W^2(Z_1'(0)\, z_0' + Z_0'(0) Z_1') =$$
$$+ Z_0^2(2WZ_0 - 2W^2) + 2Z_0(1 - W^2)(Z_1 - Z_0'(0)Z_0)$$
$$+ f_1 Z_0^2(W^2 Z_1'(0) - 2WZ_0'(0)\, Z_0)$$
$$+ f_1 Z_0'(0)\, W^2(Z_0^2 - 2Z_0'(0)\, Z_0^2 + 2Z_0 Z_1)$$
$$+ f_2 Z_0'(0)\, W^2 Z_0^2 \qquad (32a)$$

$$Z_1(0) = 0 \qquad (32b)$$

$$Z_1(1) = 0 \qquad (32c)$$

Accurate results for F as well as other unsteady flow quantities have been obtained[6] by using the above series solution for Z with no more expansions and truncations in the evaluation of the double integral (27), as was done by Messiter[4]. It is shown in Ref. 6 that such further series expansions and truncations are not valid and should be avoided. In what follows we shall solve systems (31) and (32) for parabolic and diamond cross-sections and present the results for the function F together with other existing theories for comparison. We shall see that the present results improve and extend over these theories.

III.1 Solution for wings with Parabolic Arc Section

For wings with parabolic arc cross-section the function f is given by

$$f = \nu\Omega\left(1 - \left(\frac{\tilde{z}}{\Omega}\right)^2\right) \tag{33}$$

where $\nu = \frac{c}{\sqrt{\varepsilon}\, b}$ is the thickness parameter of the wing . The functions f_1, f_2,... are given by

$$f_1 = 2\nu \ , \quad f_2 = f_3 = 0 \ ,$$

and the solutions of (31),(32) are given by

$$Z_0 = \frac{2W}{\nu_1(1+\nu_2 W^2)} \tag{34}$$

$$Z_1 = \begin{cases} Z_0^2\left[\sqrt{\nu_2}\, \mathrm{Tan}^{-1}\sqrt{\nu_2}\ (W+W^{-1}) + \dfrac{(1-6\nu)(W-W^{-1})}{2\nu_1} - 2\sqrt{\nu_2}\, \mathrm{Tan}^{-1}(\sqrt{\nu_2}\, W)\right] , \quad 0 \le \nu \le 0.5 & (35) \\ Z_0^2\left[\dfrac{1}{2}\sqrt{|\nu_2|}\ (W + W^{-1})\ell n\, \dfrac{1 - \sqrt{|\nu_2|}}{1 + \sqrt{|\nu_2|}} + \dfrac{(1-6\nu)(W-W^{-1})}{2\nu_1} - \sqrt{|\nu_2|}\ \ell n\, \dfrac{1-\sqrt{|\nu_2|}W}{1+\sqrt{|\nu_2|}W}\right] , \quad \nu \ge 0.5 \ , & (36) \end{cases}$$

where $\nu_1 = 1 + 2\nu$, $\nu_2 = \frac{1 - 2\nu}{1 + 2\nu}$.

The special case of a flat delta wing corresponding to $\nu = 0$ has been studied in detail in Refs. 4 and 7 and will not be discussed further. Only we should mention that for flat wing the third term in the series solution, Z_2 , has also been found. The results for the function F using 2 terms and 3 terms of the series solution were found to differ only slightly. This indicates the rapid convergence and thus the accuracy of the series solution.

III.2 Solution for wings with Diamond Section

In the derivation of the functional-equation (24a) it was assumed that the functions $f(\tilde{z})$ and $f'(\tilde{z})$ are continuous. Thus an immediate application of the method to diamond section (for which $f'(0)$ is undefined) is not possible. However, this apparent difficulty can be easily removed by considering the diamond section as the limiting shape of a hyperbola when it gets close to its asymptotic lines. It can be shown[6] that, the function $f(\tilde{z})$ for the hyperbola is given by

$$f(\tilde{z}) = \nu\Omega\left[1 - \sqrt{\frac{1}{A^2} + \left(\frac{\tilde{z}}{\Omega_0}\right)^2}\,\right] / \left(1 - \frac{1}{A}\right) \tag{37}$$

where A is a parameter and $\Omega_0 = A\Omega/\sqrt{A^2 - 1}$.
The functions f_1 and f_2 are given by

$$f_1 = \frac{\nu(A+1)}{[1+(A^2-1)Z_0^2]^{3/2}} \tag{38}$$

$$f_2 = \frac{-3\nu(A+1)(A^2-1)Z_0 Z_1}{[1 + (A^2-1)Z_0^2]^{5/2}} \ . \tag{39}$$

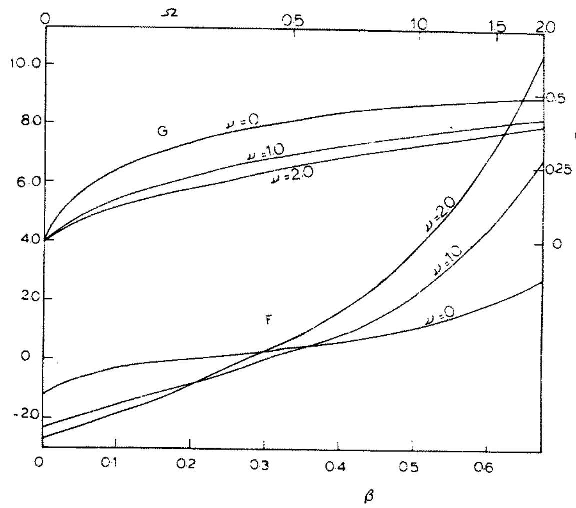

Fig. 2 The functions F and G for Parabolic Arc wing.

With special care in taking the limit $A \to \infty$, the solutions for Z_0 and Z_1 are found as follows[6]

$$Z_0 = \frac{(2-\nu)W}{1+W^2-\nu W\, \mathrm{sgn}(W)} \tag{40}$$

$$Z_1 = Z_0^2\left\{\frac{g(\nu)}{Z_0}\, \frac{2W}{(2-\nu)} - \frac{2}{W} + \nu\, \mathrm{sgn}(W)\, \ell n\, \frac{WZ_0}{(2-\nu)} + \frac{\nu\, \mathrm{sgn}(W)}{(2-\nu)} + \frac{2\nu_2}{\nu_1}\, \mathrm{sgn}(W)\left[\mathrm{Tan}^{-1}\frac{2|W|-\nu}{\nu_1} + \mathrm{Tan}^{-1}\frac{\nu}{\nu_1}\right]\right\} \tag{41}$$

where

$$g(\nu) = 3 + \nu\, \ell n(2 - \nu) - 2\,\frac{\nu_2}{\nu_1}\left(\mathrm{Tan}^{-1}\frac{2-\nu}{\nu_1} + \mathrm{Tan}^{-1}\frac{\nu}{\nu_1}\right),$$

$$\nu_1 = \sqrt{4 - \nu^2} \ , \quad \nu_2 = \nu^2 - 2 \ .$$

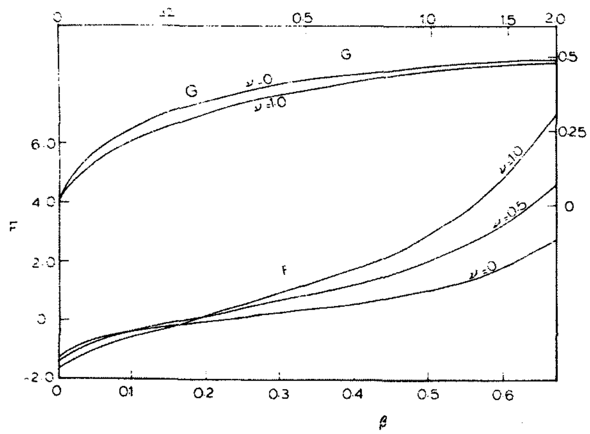

Fig. 3 Th functions F and G for Diamond wing.

Once again the flat wing solution can be recovered from (40), (41) by setting $\nu = 0$.

Having found an approximate solution to equations (24), we can now find the function F by using these solutions in equation (27). The evaluation of the double integrals involved is straightforward after making certain transformations to remove the apparent singularities (for details see Ref. 6). The results for the parabolic arc wings are shown in Fig. 2 (together with other unsteady results to be referred to in the next section) for all values of Ω and ν up to 2, and those for the diamond sections are shown in Fig. 3 for all Ω and ν up to 1. Comparisons with other theories are given in Fig. 4, 5 and 6 .

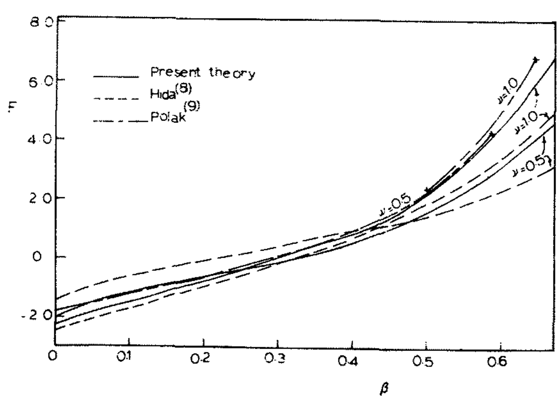

Fig. 4 Comparison between present and other theories for Parabolic Arc wing.

Figs. 2 and 3 show that the thickness of the wing tends to decrease the normal force for small values of Ω and to strongly increase it for the large values of Ω . Similar conclusions has been reached by Hida[8] . Figs. 4 and 5 show the good agreement between the present results and those of Hida for small values of Ω , while for larger Ω Hida's method gives much smaller value for F than the present theory. It is evident from Fig. 4 that Polak's[9] results for the parabolic wing agree with the present results for large Ω . Hida's method is an approximate inverse method which fails to give the correct value for F even for flat wing as $\Omega \to 0$, while the present theory predicts the exact value for flat wing. Moreover, Polak has pointed out that Hida's results for large Ω were in error and he added one more term in Hida's series solution to obtain a better approximation. This has led to good agreement between Polak's results and the present theory as shown in Fig. 4. Following Hida, Squire[10] also studied steady flow over conical delta wings using some approximate numerical methods. But no results were given for F which can be compared with the present theory. Finally, for flat delta wings (Fig.6), the present method gives almost identical results with Messiter's numerical solution of the functional-differential equation. While Messiter's numerical solution terminates at $\Omega = 0.5$ because of the increasing error in the numerical scheme, the present solution may be used for the whole range of Ω and for general conical wings.

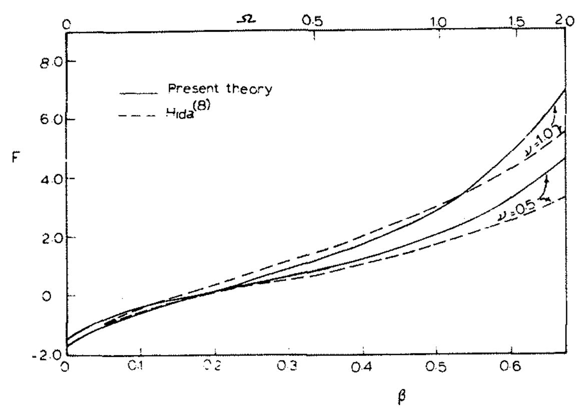

Fig. 5 Comparison between present and other theories for Diamond wing.

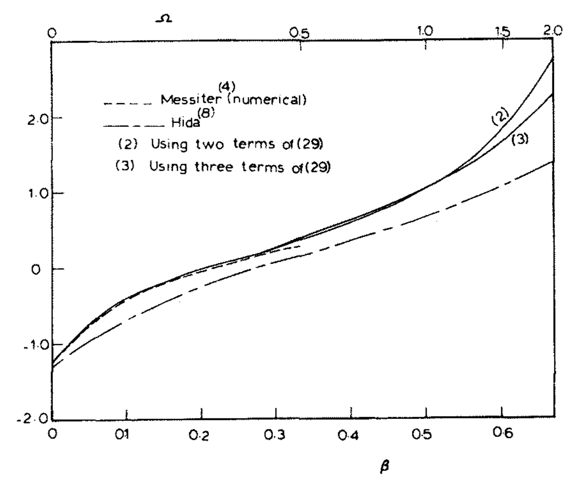

Fig. 6 Comparison between present and other theories for flat delta wings.

IV. The Unsteady Flow Solution

Now we consider the solution of system II in section 2. An important property of system II is that the unsteady pressure P_1 is decoupled from the other functions (equations (16c) and (18d)) and can be easily found to be

$$P_1 = \cot\alpha(y-y_s)[k^2(h-x) + 2ik] + 2\cot\alpha[ik(h-x)-1], \quad (42)$$

the other functions are not of interest in the evaluation of the aerodynamic derivatives of the wing and will not be discussed here.

With the above simple closed form solution for the unsteady pressure $P_1(x,y,z)$, we can derive closed form formulas for the aerodynamic derivatives. The pitching moment coefficient C_m is given by

$$C_m = \frac{\bar{M}}{\frac{1}{2}\bar{\rho}_\infty \bar{U}^2 \bar{\ell}^2 \bar{b}} = 4\int_0^{\bar{\ell}}\int_0^{b\bar{x}} (\bar{x}-h\bar{\ell})[\bar{p}(\bar{x},\bar{y}_b,\bar{z},t) - \bar{p}_0]\, d\bar{z}\, d\bar{x}/(\bar{\rho}_\infty \bar{U}^2 \bar{\ell}^2 \bar{b}) \quad (43)$$

where $\bar{M}$ is the pitching moment, and $\bar{p}_0$ the value of $\bar{p}$ when the wing is stationary and its leading edges are in the plane $\bar{y} = 0$. Introducing the conical coordinate $\tilde{z}$ in (43) and let $P_1(x,y_b,z) = \tilde{P}_1(x,\nu\Omega x f(\tilde{z}),\tilde{z})$, we get

$$C_m = \frac{4}{b}\varepsilon^{\frac{1}{2}}\lambda_0 \sin^2\alpha \tan\alpha\, e^{ikt}\int_0^1\int_0^\Omega x(x-h)\cdot \tilde{P}_1(x,\nu\Omega x f(\tilde{z}),\tilde{z})\, d\tilde{z}\, dx \quad . \quad (44)$$

Since

$$C_m = \lambda_0 e^{ikt}(m_\theta + ikm_{\dot\theta}) \quad (45)$$

where $-m_\theta$ and $-m_{\dot\theta}$ are respectively the in-phase and the out-of-phase components of the aerodynamic derivatives, we get the following formulas for $-m_\theta$, and $-m_{\dot\theta}$:

$$-m_\theta = 2\sin2\alpha[\tfrac{2}{3} - h + k^2 G(\Omega,\nu)(\tfrac{1}{2}h - \tfrac{1}{3}h^2 - \tfrac{1}{5})] \quad (46)$$

$$-m_{\dot\theta} = 2\sin2\alpha[(\tfrac{1}{2} - \tfrac{2}{3}h)\, G(\Omega,\nu) + h^2 - \tfrac{4}{3}h + \tfrac{1}{2}] \quad (47)$$

where

$$G(\Omega,\nu) = \frac{1}{\Omega}\int_0^\Omega (y_s - f)\, d\tilde{z} \, , \quad (48)$$

is average shock height measured from wing surface and y_s is given by (26). Using (26) and (48) we get the following formula for G:

$$G(\Omega,\nu) = \beta\int_0^1 Z'(W)\cdot I_1 dW \quad (49)$$

and I_1 given by (28a).

Results for the function G are shown in Fig. 2 for parabolic wings and in Fig. 3 for diamond wings, for all the range of Ω and various values of the thickness parameter ν.

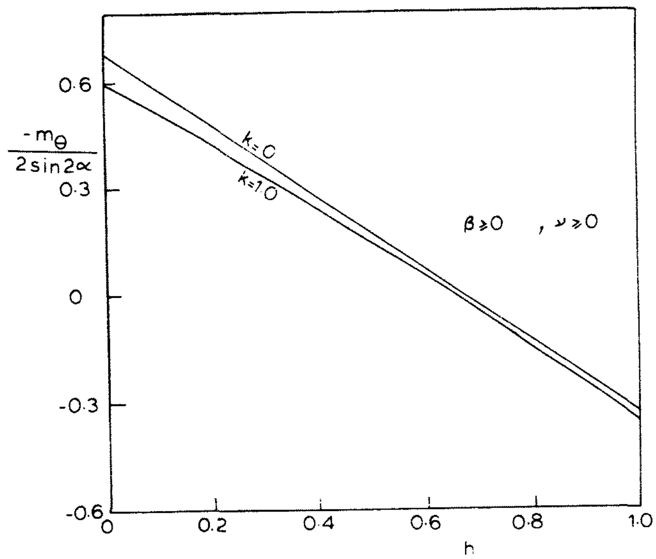

Fig. 7 The in-phase aerodynamic derivative for Diamond and Parabolic Arc Delta wings.

Using the results for $G(\Omega,\nu)$ we now discuss the aerodynamic derivatives of a pitching delta wing at high angles of attack with detached shock wave as follows. Although both the in-phase derivative $-m_\theta$ and the out-of-phase derivative $-m_{\dot\theta}$ in general may be infinite series in k^2, it turns out in the present case that $-m_{\dot\theta}$ is independent of k while $-m_\theta$ includes only terms up to k^2. Fig. 7 shows that $-m_\theta$ changes only slightly with k for k up to 1.0, for both parabolic and diamond sections having $\nu = 0.5$.

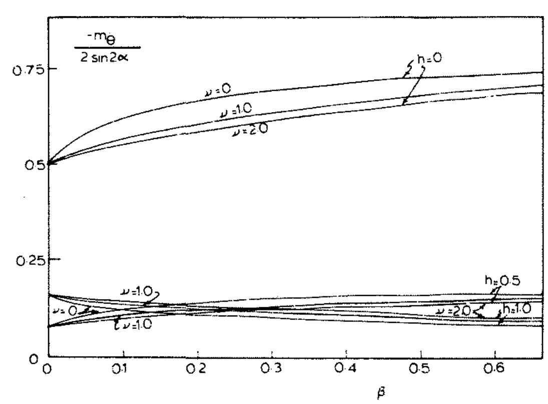

Fig. 8 The out-of-phase aerodynamic derivative for Parabolic Arc Delta wings.

Equations (46) and (47) show that $-m_\theta/2\sin2\alpha$ and $-m_{\dot\theta}/2\sin2\alpha$ do not depend on the flight parameters M_∞,α,γ and wing aspect ratio b, thickness c and shape f separately, but rather depend on their combination as represented by the function $G(\Omega,\nu)$ showing the importance of the parameters Ω, and ν in analysing the aerodynamic stability of a delta wing. Equation (46) shows that the principle part of $-m_\theta/2\sin2\alpha$ i.e. when $k = 0$, does not depend on any parameter and that the aerodynamic centre is at $h = \frac{2}{3}$ whether the wing is flat or curved.

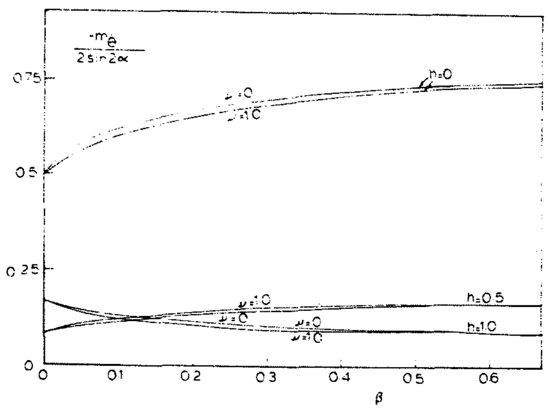

Fig. 9 The out-of-phase aerodynamic derivative for Diamond Delta wings.

Fig. 8 shows that both the aspect-ratio (represented by Ω) and thickness (represented by ν) of the parabolic wings have moderate effects on the derivative $-m_{\dot\theta}$ and that the thickness effect increases as the axis of oscillation gets closer to wing apex. Fig. 9 shows that for the diamond section the effect of aspect-ratio on $-m_{\dot\theta}$ is moderate but the thickness effect is negligible. Fig. 10, the last figure, shows that for both the parabòlic and diamond sections, the derivative $-m_\theta$ is almost independent of both the parameters Ω and ν. Similar conclusions have been reached by Liu and Hui[3] for pitching flat delta wings with attached shock waves.

Finally the out-of-phase aerodynamic derivative $-m_{\dot\theta}$ may be re-written

$$\frac{-m_{\dot\theta}}{2\sin2\alpha} = \left[h - \frac{1}{3}(G + 2)\right]^2 + \frac{1}{18}(1-G)(1+2G) \quad (50)$$

But Figs. 2 and 3 show that $G < 1$ for $0 < \Omega < 2$ and various values of ν. We therefore conclude that within the thin shock layer approximation, the pitching motion of a slender delta wing having conical thickness and detached shock wave in hypersonic flight is always stable dynamically. In this context it is noted that the pitching motion of a flat delta wing with attached shock wave[3] and that of a wedge[2], may become aerodynamically unstable if the flight Mach number is low enough.

V. Conclusions

A general and a systematic study of steady and unsteady flow past a convex slender conical delta wing at large angles of attack with detached shock waves is presented using thin shock layer theory. In the steady flow part an extension and improvement over Messiter's and other existing theories is given. In particular the effect of wing thickness and curvature on the normal force is found. For the unsteady part, closed form simple analytical expressions for the aerodynamic derivatives of the wing are found and used to establish a criterion for the stability of flight of delta wings. It has been shown that pitching delta wings are always stable dynamically in hypersonic flight. To the authors' knowledge there exists no analytical or experimental results to compare with for the unsteady theory.

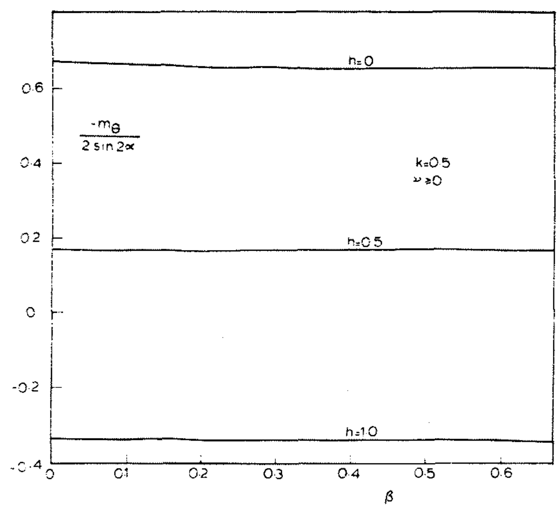

Fig. 10 The in-phase aerodynamic derivative for Parabolic and Diamond Delta wings.

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